

Arithmetic Progression

Important Terms

1. A **sequence** is an arrangement of numbers in a definite order according to some rule.
2. The various numbers occurring in a sequence are called its **terms**. We denote the terms of a sequence by $a_1, a_2, a_3 \dots$ etc. Here, the subscripts denote the positions of the terms. In general, the number at the n th place is called the n th term of the sequence and is denoted by a_n . The n th term is also called the **general term** of the sequence.
3. A sequence having a finite number of terms is called a **finite sequence**.
4. A sequence which do not have a last term and which extends indefinitely is known as an **infinite sequence**.

Arithmetic Progression

An **arithmetic progression** is a list of numbers in which each term is obtained by adding a fixed number to the preceding term, except the first term.

Each of the numbers of the sequence is called a **term** of an Arithmetic Progression. The fixed number by which the successive terms differ is called the **common difference**. This common difference could be a positive number, a negative number or even zero.

A list of numbers a_1, a_2, a_3, \dots is an A.P., if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3 \dots$ give the same value, i.e., $a_{k+1} - a_k$ is same for all different values of k .

The **general form of an A.P.** is $a, a + d, a + 2d, a + 3d, \dots$

If the A.P $a, a + d, a + 2d, \dots, \ell$ is reversed to $\ell, \ell - d, \ell - 2d, \dots, a$, then the common difference changes to negative of the common difference of the original sequence.

General Term of an A.P.

The general term of an A.P. is given by

$$t_n = a + (n - 1)d$$

where 'a' is the first term and d is the common difference.

Sum of n terms of an A.P.

$$S = \frac{n}{2}[2a + (n - 1)d] \quad \text{OR} \quad S = \frac{n}{2}[a + \ell]$$

where 'a' is the first term, d is the common difference, l is the last term and n is the total number of terms.

The n^{th} term of an A.P is the difference of the sum to first n terms and the sum to first (n - 1) terms of it. That is, $t_n = S_n - S_{n-1}$

Three or More Terms in A.P.

Sometimes we require certain number of terms in *A.P.* The following ways of selecting terms are generally very convenient.

Number of terms	Terms	Common difference
3	$a - d, a, a + d$	d
4	$a - 3d, a - d, a + d, a + 3d$	$2d$
5	$a - 2d, a - d, a, a + d, a + 2d$	d
6	$a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$	$2d$

It should be noted that in case of an odd number of terms, the middle term is 'a' and the common difference is 'd' while in case of an even number of terms the middle terms are $a - d, a + d$ and the common differences is $2d$.

Arithmetic Mean

If three number a, b, c (in order) are in A.P. Then,

$$b - a = \text{Common difference} = c - b$$

$$\Rightarrow b - a = c - b$$

$$\Rightarrow 2b = a + c$$

Thus a, b, c are in A.P., if and only if $2b = a + c$. In this case, b is called the **arithmetic mean** of a and c .