Compound Interest

Important concepts of Compound Interest

1. The money is said to be compounded, if the interest at the end of a year or some other fixed period which has fallen due is not paid to lender but is added to the principal, so that the amount at the end of this period becomes the principal for the next period. This process is repeated until the amount for the last period has been obtained.
2. Some entities such as population, the height of a tree, weight and height of a child, increase in magnitude over a period of time. The function of increase is called growth. Growth per unit of time is called the rate of growth.

Important Formulae:

1. Simple Interest $S.I. = \frac{P \times R \times T}{100}$, where P is the principal, R the rate of interest and T be the time.
2. When interest is compounded annually. Let P be the principal, r be the rate of interest, n be the time then

$$A = P\left(1 + \frac{r}{100}\right)^n$$

and $C.I. = \text{Amount} - \text{Principal}$

3. In case of simple interest, the principal remains the same for the whole time.

More about Compound Interest

1. In case of compound interest, the principal increases
   i. after every year, if compounded annually
   ii. after every six months, if compounded half yearly
   iii. after every four months, if compounded quarterly

2. On the same sum and at the same rate of interest compounded yearly,
   i. the C.I of 2nd year is always more than the C.I of 1st year.
   ii. C.I. of 3rd year is more than C.I of 2nd year.

3. On the same sum and at the same rate of interest compounded half-yearly,
   i. the C.I of 2nd half year is always more than the C.I of 1st half-year.
   ii. C.I. of 3rd half-year is more than C.I of 2nd half-year.

4. In general, for any period, the C.I is more than the C.I. of the previous period.

5. The difference between the compound interests for any two consecutive conversion periods (year or half-year) is the interest of one period on the C.I of the preceding conversion period.

6. The difference between the amounts for any two consecutive conversion periods is also the interest of one period on the amount of the preceding period.

7. When interest is compounded annually but rates being different for different years. Let *P* be the principal, time to be 2 years and r_1% & r_2% be the rates of interest for first year & second year respectively.

   Then Amount after 2 years

$$A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)$$

8. When interest is compounded half-yearly: Let *P* be the principal r% be the rate of interest and n years be the time.

$$A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}$$

9. When interest is compounded quarterly: Let *P* be the principal r% be the rate of interest and n years be the time.

$$A = \left(1+\frac{r}{4\times 100}\right)^{n\times 4}$$

10. Let us study the following table:

Number of years	Compounded Yearly	Compounded half - yearly
$n = 1$	$A = P\left(1+\frac{r}{100}\right)^{1}$	$A = P\left(1+\frac{r}{2\times 100}\right)^{1\times 2}$
$n = 1\frac{1}{2}$	$A = P\left(1+\frac{r}{100}\right)^{1}\cdot\left(1+\frac{r}{2\times 100}\right)^{\frac{1}{2}\times 2}$	$A = P\left(1+\frac{r}{2\times 100}\right)^{\frac{3}{2}\times 2}$
$n = 2$	$A = P\left(1+\frac{r}{100}\right)^{2}$	$A = P\left(1+\frac{r}{2\times 100}\right)^{2\times 2}$
$n = 2\frac{1}{2}$	$A = P\left(1+\frac{r}{100}\right)^{2}\cdot\left(1+\frac{r}{2\times 100}\right)^{\frac{1}{2}\times 2}$	$A = P\left(1+\frac{r}{2\times 100}\right)^{\frac{5}{2}\times 2}$

Applications of compound interest

1. Let P be the population of a city or town at the beginning of a certain year and the population growth rate is R% per annum, then

$$\text{Population after N years} = A = P\left(1 + \frac{R}{100}\right)^N$$

2. Let P be the population of a city or town at the beginning of a certain year and the population depreciates at the rate of R% per annum, then

$$\text{Population after N years} = A = P\left(1 - \frac{R}{100}\right)^N$$

3. Let P be the population of a city or town at the beginning of a certain year. If the population growth rate is R_1% during first year and R_2%during second year , R_3%during third year and so on for N years

$$\text{Population after N years} = A = P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right).....\left(1 + \frac{R_N}{100}\right)$$