Factorisation

Important concepts

1. When a polynomial is the product of two or more polynomials, each of the later polynomials is called its factors.
2. The method of expressing a given polynomial as a product of two or more polynomials is called factorization.
3. If a given polynomial contains a common factor which may be either a constant or a variable, we divide each term separately by this factor.

   Example: Each term of the expression $3a^2 - 9ab$ has 3a as a common factor.

   Therefore, we have $3a^2 - 9ab = 3a(a - 3b)$

4. If the polynomial has even number of terms, then the terms are first arranged in groups such that each group has a common factor.

   Example: Factorise : $a^3 + a - 3a^2 - 3$

$$
\begin{aligned}
a^3 + a - 3a^2 - 3 &= (a^3 - 3a^2) + (a - 3) && [\text{Forming groups}] \\
&= a^2(a - 3) + 1(a - 3) && [\text{Taking out common factors from each group}] \\
&= (a - 3)(a^2 + 1) && [\text{Taking } (a - 3) \text{ common}]
\end{aligned}
$$

5. If the polynomial is trinomial in nature, i.e. , it has 3 terms ,we first arrange the terms in descending order. Then, split the middle term in such a way that the product is equal to the product of first and the last term.

   Example : Factorise $a^2 + 10a + 24$

$$
\begin{aligned}
a^2 + 10a + 24 &= a^2 + 6a + 4a + 24 && [\text{since } 6 + 4 = 10 \text{ and } 6 \times 4 = 24] \\
&= a(a + 6) + 4(a + 6) && [\text{Taking out common factors from each group}] \\
&= (a + 6)(a + 4) && [\text{Taking } (a + 6) \text{ common}]
\end{aligned}
$$

6. Factorisation using difference of two squares: $x^2 - y^2 = (x + y)(x - y)$

Example : Factorise $9a^2 + 3a - 8b - 64b^2$

$$\begin{aligned} 9a^2 + 3a - 8b - 64b^2 &= 9a^2 - 64b^2 + 3a - 8b \quad [\text{Forming groups}] \\ &= (3a)^2 - (8b)^2 + 3a - 8b \quad [\text{Rewriting}] \\ &= (3a - 8b)(3a + 8b) + (3a - 8b) \quad \begin{bmatrix} \text{Writing as difference} \\ \text{of squares} \end{bmatrix} \\ &= (3a - 8b)(3a + 8b + 1) \quad [\text{Taking } (3a - 8b) \text{ common}] \end{aligned}$$

7. Sum of Difference of Two Cubes:

1. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$