Geometric Progression

Important Terms

1. A **sequence** is an arrangement of numbers in a definite order according to some rule.

2. The various numbers occurring in a sequence are called its **terms**. We denote the terms of a sequence by a_1, a_2, a_3*…etc.* Here, the subscripts denote the positions of the terms. In general, the number at the *n*th place is called the *n*th term of the sequence and is denoted by a_n. The *n*th term is also called the **general term** of the sequence.

3. A sequence having a finite number of terms is called a **finite sequence**.

4. A sequence which do not have a last term and which extends indefinitely is known as an **infinite sequence**.

Geometric Progression

A sequence is said to be in **geometric progression** or G.P., if the ratio of any term to its preceding term is same throughout. Constant Ratio is **common ratio** denoted by **'r'**.

General Term of a G.P.

The general term of a G.P. is given by

$$\mathbf{t_n = ar^{n-1}}$$

where 'a' is the first term and 'r' is the common ratio.

Properties of G.P.

1. The ratio between the consecutive terms of a G.P. is always the same.

   $$\Rightarrow \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \ldots\ldots\ldots$$

2. r^{th} term from the beginning $\times$ r^{th} term from the end = constant = First term $\times$ Last term

3. If a, b and c are in G.P,

   $$\Rightarrow \frac{b}{a} = \frac{a}{c} \Rightarrow b^2 = ac$$

4. In a G.P. if the terms at equal distances are taken, these terms are also in G.P.

5. If each term of a G.P. be multiplied or divided by the same non-zero number, the resulting series is also a G.P.

6. The series obtained by taking the reciprocals of the terms of a G.P. is also a G.P.

7. If each term of a G.P. is raised to the same non-zero number, the resulting series is also a G.P.

8. If the corresponding terms of two different G.P.s are multiplied/divided together, the resulting series, so obtained, is also a G.P.

Sum of n terms of a G.P.

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$

Case I: r = 1

$S_n = a + a + a + \ldots +$ to n terms $= na$

Case II: r < 1

$$S_n = \frac{a(1-r^n)}{1-r}$$

Case III: r > 1

$$S_n = \frac{a(1-r^n)}{1-r}$$

Sum of Infinite terms of a G.P.

$$\text{Sum of infinite terms in G.P.} = \frac{a}{1-r}, \text{ if } |r| < 1$$

Geometric Mean Between Numbers a and b

If a and b are two positive numbers then a, G, b are in G.P.

$$\Rightarrow G^2 = ab$$
$$\Rightarrow G = \sqrt{ab}$$

Three or More Terms in G.P.

Number of terms is three: $\frac{a}{r}$, a, ar

Number of terms is four: $\frac{a}{r^3}, \frac{a}{r}$, a, ar^3

Number of terms is five: $\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2

and so on.