Linear Inequations

Introduction to inequations

1. An inequation is a mathematical statement which states that one quantity is not equal to the other.
2. In an inequation, the signs, $'<','>','\leq'$ and $'\geq'$ are called signs of inequality.

Rules for solving inequations algebraically:

1. On transferring a positive term (the term in addition) from one side of an inequation to its other side, the sign of the term becomes negative.
   Example: $5x + 3 \leq 13 \Rightarrow 5x \leq 13 - 3$

2. On transferring a negative term from one side of an inequation to its other side, the sign of the term becomes positive.
   Example: $5x - 3 \leq 13 \Rightarrow 5x \leq 13 + 3$

3. If each term of an inequation be multiplied or divided by the same positive number, the sign of inequality remains the same.
   Case I: If p is positive and x < y

   $x < y \Rightarrow px < py$ and $\left(\dfrac{x}{p}\right) < \left(\dfrac{y}{p}\right)$,

   $x > y \Rightarrow px > py$ and $\left(\dfrac{x}{p}\right) > \left(\dfrac{y}{p}\right)$,

   $x \leq y \Rightarrow px \leq py$ and $\left(\dfrac{x}{p}\right) \leq \left(\dfrac{y}{p}\right)$,

   $x \geq y \Rightarrow px \geq py$ and $\left(\dfrac{x}{p}\right) \geq \left(\dfrac{y}{p}\right)$.

   Example: $2x \leq 5 \Rightarrow 2 \times 2x \leq 2 \times 5 \Rightarrow 4x \leq 10$

4. If each term of an inequation be multiplied or divided by the same negative number, the sign of inequality reverses.
   Case II: If p is negative

   $x < y \Rightarrow px > py$ and $\left(\dfrac{x}{p}\right) > \left(\dfrac{y}{p}\right)$

   $x \geq y \Rightarrow px \leq py$ and $\left(\dfrac{x}{p}\right) \leq \left(\dfrac{y}{p}\right)$

   Example: $x \leq -2 \Rightarrow (-3) \times x \geq (-3) \times (-2) \Rightarrow -3x \geq 6$

5. If sign of each term on both the sides of an inequation is changed, the sign of inequality gets reversed.
   Example: $-3x \geq 20 \Rightarrow 3x \leq -20$

6. If both the sides of an inequation are positive and both are negative, then on taking their reciprocals, the sign of inequality reverses.
   Example: If x and y both are either positive or both are negative, then

   $$x > y \Leftrightarrow \frac{1}{x} < \frac{1}{y}$$

   $$x \le y \Leftrightarrow \frac{1}{x} \ge \frac{1}{y}$$

7. The sign of the inequality does not change when a positive number is added or subtracted from both sides.
   i. If x<y, then x+a <y+a.
   ii. If x<y, then x-a < y-a.

Replacement Set and Solution Set

1. Replacement set is the set from which the values of the variable involved in the inequation are chosen.

2. Solution set is the subset of the replacement set, whose elements satisfy the given inequation.
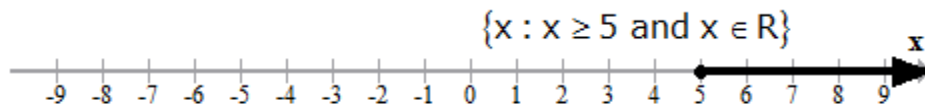   Example: Let the given inequation be $x \le 4$, if:
   i. the replacement set = N, the set of natural numbers;
      the solution set = {1, 2, 3, 4}
   ii. the replacement set = W, the set of whole numbers;
      The solution set = {0, 1, 2, 3, 4}
   iii. the replacement set = Z or I, the set of Integers;
      The solution set = {….-2, -1, 0, 1, 2, 3, 4}
   iv. the replacement set = R, the set of real numbers;
      The solution set = $\{x : x \in R \text{ and } x \le 4\}$

3. The solution set depends on the replacement set.
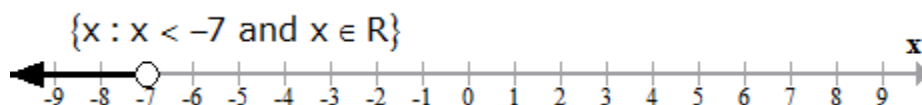
Representation of the solution on the number line:

  i. The solution set of an inequation can be represented on a real number line.

  ii. A hollow circle marks the end of a range with a strict inequality $(< \text{ or } >)$.

  iii. A darkened circle marks the end of a range involving an equality as well $(\leq \text{ or } \geq)$.
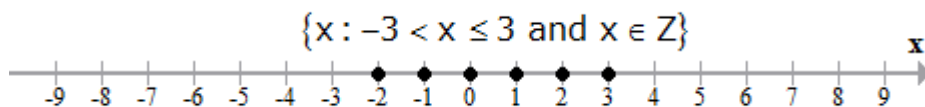
  iv. Some examples:

1.

$$\{x : x \geq 5 \text{ and } x \in R\}$$

2.

$$\{x : x < -7 \text{ and } x \in R\}$$

3.

$$\{x : -3 < x \leq 3 \text{ and } x \in Z\}$$

4.

$$\{x : -3 < x \leq 5 \text{ and } x \in R\}$$