Matrices

Important Terms

1. A set of mn numbers arranged in the form of a rectangular array of m rows and n columns is called an m x n matrix

2. Each number or entity in a matrix is called its element.

3. In a matrix, the horizontal lines are called rows, whereas the vertical lines are called columns.

4. If a matrix contains m rows and n columns then it is said to be a matrix of order $m \times n$ (read as m by n).

5. The total number of elements in a matrix is equal to the product of its number of rows and number of columns.

6. A matrix having only one row is known as a row matrix.

7. A matrix having only one column is known as a column matrix.

8. A matrix which has an equal number of rows and columns is called a square matrix.

9. Rectangular matrix: A matrix in which number of rows are not equal to the number of columns is called a rectangular matrix.

10. A matrix each of whose elements is zero is called a zero matrix or a null matrix.

11. A square matrix which has every non–diagonal element as zero is called a diagonal matrix.

12. A diagonal matrix in which each element of its leading diagonal is unity is called identity matrix.

13. Two matrices are said to be equal, if they are of the same order and have the same corresponding elements.

14. If A is a matrix, then its transpose is obtained by interchanging its rows and columns. Transpose of a matrix A is denoted by A^t.

Operations on matrices

1. Addition of Matrices:
Let A and B be two matrices each of order m × n. Then their sum A + B is a matrix of order m × n and is obtained by adding the corresponding elements of A and B.

2. Subtraction of Matrices:
Let A and B be two matrices each of order m × n. Then their difference A – B is a matrix of order m × n and is obtained by subtracting the corresponding elements of A and B.

3. In addition or subtraction of the matrices, the order of the resulting matrix is the same as the order of matrices added or subtracted.

4. Matrix addition is commutative

i.e., A + B=B + A

5. Matrix addition is associative for any three matrices A, B and C.

A + (B + C)=(A + B) + C.

6. If A and B are two matrices, $A + X = B \Rightarrow X = B - A$

7. Additive Identity: If any matrix is added to null (zero) matrix of the same order, or a null matrix is added to a matrix of the same order, the matrix remains unaltered and hence, the null matrix is said to be the additive identity in matrices.
A null matrix is identity element for addition
i.e., A + 0 = A = 0 + A.

8. If A and B are two matrices of the same order such that:
A + B = B + A = a null matrix,
then A is said to be the additive inverse of B and B is said to be the additive inverse of A.
Additive inverse of a matrix A is its negative A.

9. If O is the null or zero matrix of the same order as matrix A, then
A + (–A) = (–A) + A = O

10. Let A and B are two matrices of the same order such that
A + X =B, where X is an unknown matrix; then X = B – A and the order of matrix X is same as that of A and B.

11. The multiplication of a matrix A by a number k gives a matrix of the same order as A, in which all the elements are k times the elements of A.

12. The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B.

13. Let A = $[a_{ij}]$ be an m × n matrix and B = $[b_{jk}]$ be an n × p matrix. Then the product of the matrices A and B is the matrix C of order m × p.

14. To get the $(i, k)^{th}$ element c_{ik} of the matrix C, we take the i^{th} row of A and k^{th} column of B, multiply them element wise and take the sum of all these products.

15. Matrix multiplication is not commutative. In general, $AB \neq BA$.

16. The product of two non-zero matrices can be a zero matrix.

17. Let A, B and C are matrices.

Then $AB = AC, A \neq 0 \Rightarrow$ it is not necessary that B=C.

In general, cancellation law is not applicable in matrix multiplication.

18. Identity matrix: The unit matrix I is known as the identity matrix for multiplication.

Let A be any square matrix and I be the unit matrix of same order, then,

$A \times I = I \times A = A$.

19. Let A, B and C be any three matrices.

Then $(AB)C = A(BC)$

Thus, matrix multiplication is associative.

20. Let A and B are two matrices.

Then $AB = 0 \Rightarrow$ it is not necessary that A=0 or B=0

21. Let A and B are two matrices.

Then If A=0 or B=0, then AB=0=BA

22. Let A, B and C be any three matrices.

(1) then $A(B + C) = AB + AC$ and $A(B - C) = AB - AC$

(2) then $(A + B)C = AC + BC$ and $(A - B)C = AC - BC$

In general, matrix multiplication is distributive over addition and subtraction.

23. Laws of algebra are not applicable to matrices.

That is,

$(A+B)^2 \neq A^2 + 2AB + B^2$

and

$(A+B)(A-B) \neq A^2 - B^2$