Mensuration: Cylinder, Cone and Sphere

1. **Right Circular Cylinder**

   A solid having uniform circular cross-section is called a cylinder.

   If r and h respectively denote the radius of the circular cross-section and the height of the cylinder, then

   Area of cross-section = πr^2

   Perimeter of cross-section = $2\pi r$

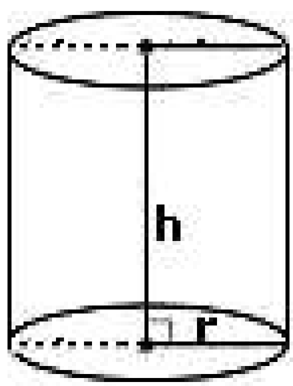
   Area of curved surface or lateral surface area = Perimeter of cross-section x Height = $2\pi rh$

   Total surface area = Curved surface area + 2(Area of cross-section)

   = $2\pi rh + 2\pi r^2$

   = $2\pi r(h + r)$

   Volume = Area of cross-section x Height = $\pi r^2 h$



2. **Right Circular Hollow Cylinder**

   If R and r respectively denote the external and internal radii of a right circular hollow cylinder and h denotes its height, then

   Thickness of its wall = R – r

   Area of cross-section = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

   External curved surface = $2\pi Rh$

   Internal curved surface = $2\pi rh$

   Total surface area = External curved surface area + Internal curved surface area

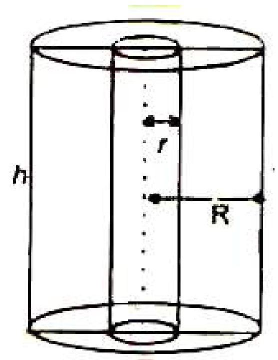
   + 2(Area of cross section)

   = $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

Volume of material = External volume – Internal volume

$= \pi R^2h - \pi r^2h)$

$= \pi(R^2 - r^2)h$



3. **Right circular cone:**

If r, h and l respectively denote the radius, height and slant height of a right circular cone, then

Slant height (l) = $\sqrt{h^2 + r^2}$

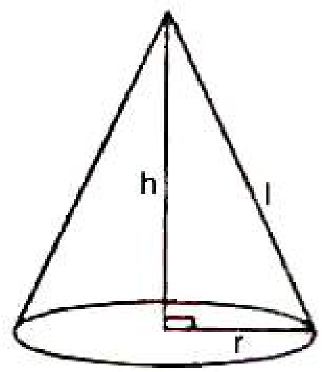
Area of curved surface = πrl

Total surface area = Area of curved surface + Area of base

$= \pi rl + \pi r^2$

$= \pi r(l + r)$

Volume = $\frac{1}{3}\pi r^2 h$

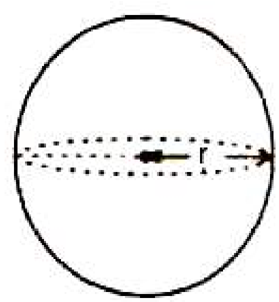


4. **Sphere**

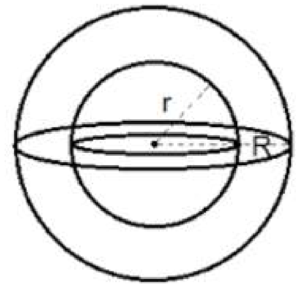
If r is the radius of the sphere, then

Surface area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$



5. **Spherical shell**

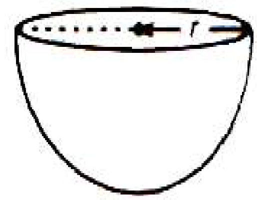


If R is the external radius and r is the internal radius of a spherical shell, then

Surface area (outer) = $4\pi R^2$

Volume of material = $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(R^3 - r^3\right)$

6. **Hemisphere**



If r is the radius of the hemisphere, then

$$\text{Total surface Area} = \frac{1}{2} \times \text{Surface area of sphere} + \text{ Area of base}$$
$$= \frac{1}{2} \times 4\pi r^2 + \pi r^2$$
$$= 3\pi r^2$$

$$\text{Volume} = \frac{1}{2} \times \text{Volume of sphere}$$
$$= \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$= \frac{2}{3}\pi r^3$$

Conversion of Solids

When a solid is melted and converted to another, volume of both the solids remains the same, assuming there is no wastage in the conversions.

However, the surface area of the two solids may or may not be the same.

Combination of Solids

The total surface area of the solid formed by the combination of solids is the sum of the curved surface area of each of the individual solids.

The volume of the solid formed by the combination of basic solids is the sum of the volumes of each of the basic solids.