

Ratio and Proportion

Important Concepts-Ratio

1. The ratio between x and y is the fraction $\frac{x}{y}$ and written as $x : y$, where, $y \neq 0$
Here, the first quantity x is known as Antecedent and second quantity y is known as Consequent.
2. The ratio $\frac{x}{y}$ has no unit.
3. Ratio is taken only between positive quantities.
4. Ratio exists only between two quantities of the same kind.
5. The value of a ratio remains unchanged, if both the antecedent (N^r) and the consequent (D^r) are multiplied or divided by the same quantity.
6. The ratio is in its lowest terms if the H.C.F of its both the terms is unity, 1.
7. Ratio $a:b$ is not equal to $b:a$, unless $a=b$
8. If a quantity increases or decreases in the ratio $a:b$, the new resulting quantity is equal to $\frac{b}{a}$ times of the original quantity.
9. Compounded Ratio is the ratio obtained by multiplying two or more ratios together
Let $a : b$ and $x : y$ be the two ratios then their compounded ratio be
 $\frac{a}{b} \times \frac{x}{y} = \frac{ax}{by}$ i.e., $ax : by$.
Compounded ratio of $a:b$, $c:d$ and $e:f$ is $ace: bdf$
10. Duplicate ratio is the compound ratio of two equal ratios.
Duplicate ratio of $x : y$ is $x^2 : y^2$.
11. Triplicate ratio is the compound ratio of three equal ratios.
Triplicate ratio of $x : y$ is $x^3 : y^3$.
12. Sub-duplicate ratio of $x : y$ is $\sqrt{x} : \sqrt{y}$.
The sub-triplicate ratio of $x : y$ is $x^{1/3} : y^{1/3}$.
13. The reciprocal ratio is the ratio between the reciprocals of two quantities.
The reciprocal ratio of $x : y$ is $\frac{1}{x} : \frac{1}{y}$ i.e., $y : x$.

Important Concepts-Proportion

1. $a : b :: c : d \Leftrightarrow \frac{a}{b} = \frac{c}{d}$.

This is called cross product rule.

If four quantities a, b, c and d are such that the ratio a:b is equal to the ratio c:d then we say a, b, c and d are in proportion. We express it by writing $a:b :: c:d$. Here, a and d are called extreme terms while b and c are called the middle terms. Here, d is called the fourth proportional.

2. Three numbers a, b and c are said to be in continued proportion if $a:b::b:c$ that is $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$.

3. If a, b and c are in continued proportion then b is called the mean proportional or geometric mean of a and c. Thus, $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac \Rightarrow b = \sqrt{ac}$.

4. If a, b, c, d, e, f, ... are in continued proportion if and only if, $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \dots$

5. If a, b and c are in continued proportion, then 'a' is called the first proportional.

6. If a, b and c are in continued proportion, then 'c' is called the third proportional.

Some properties of Ratio and Proportion

1. Invertendo: If $a:b:c:d$ then $b:a::d:c$, that is $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$

2. Alternendo: If $a:b:c:d$ then $a:c::b:d$, that is $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$

3. Componendo: If $a:b:c:d$ then $(a + b):b :: (c + d):d$, that is $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$

4. Dividendo: If $a:b:c:d$ then $(a - b):b :: (c - d):d$, that is $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$

5. Componendo and Dividendo: If $a:b:c:d$ then

$(a + b):(a - b)::(c + d):(c - d)$, that is $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

6. Convertendo: If $a:b:c:d$ then $a:(a - b)::c:(c - d)$, that is $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{a-b} = \frac{c}{c-d}$

7. For any two or more equal ratios, each ratio is equal to the ratio between sum of their antecedents and sum of their consequents.

Therefore, we have,

(i) $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+c}{b+d}$

(ii) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \Rightarrow \frac{a+c+e}{b+d+f}$