Rational and Irrational Numbers

Rational Numbers

1. A rational number is a number that can be expressed in the form *a/b* where
2. *a* and *b* are integers, and $b \neq 0$. The set of rational number is denoted by Q. Thus
   $Q = \left\{\frac{a}{b} : a, b \in Z, b \neq 0\right\}$
3. Between two rational numbers, there always exists infinite rational numbers.
4. Every integer (positive, negative or Zero) and every decimal number is a rational number.
5. All terminating and non-terminating but repeating (or periodic or recurring) decimals are rational numbers.
6. Let $x=\frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which terminates.
7. Every rational number can be expressed either as a terminating decimal or as a recurring decimal.

Irrational Numbers

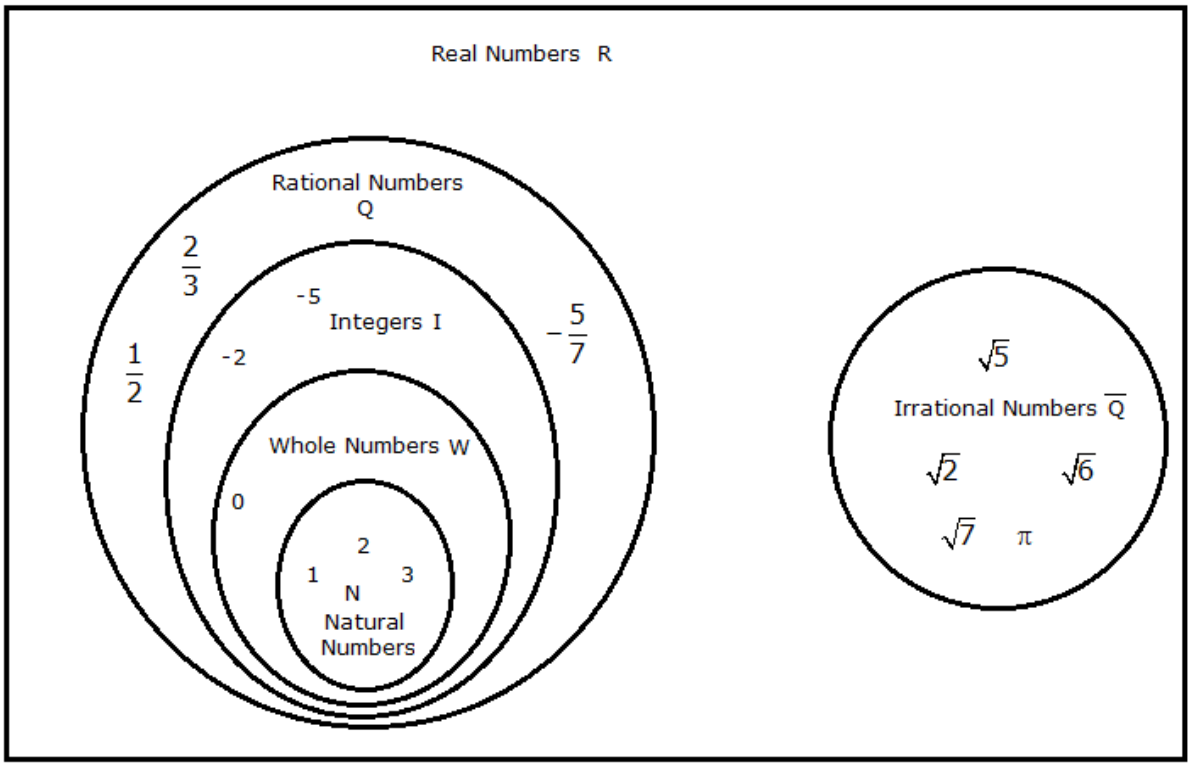
1. The real numbers, which are not rational, are called irrational numbers.
2. The square roots, cube roots etc. of natural numbers are irrational numbers; if their exact value can not be obtained.
3. All non-terminating non-repeating decimals are irrational numbers.
4. π is an irrational number.
5. If m is not a perfect square, the $\sqrt{m}$ is irrational.
6. If a and b are two positive numbers such that ab is not a perfect square, then
   1. A rational number between a and b is $\frac{a+b}{2}$
   2. An irrational number between a and b is $\sqrt{ab}$

More about Irrational Numbers

1. For any two positive rational numbers x and y,
   $\sqrt{x^2} = x, \sqrt{y^2} = y, \sqrt{xy} = \sqrt{x} \cdot \sqrt{y}, \sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$
   If $\sqrt{x}$ and $\sqrt{y}$ are irrational numbers, then, $\sqrt{x} > \sqrt{y} \Rightarrow x > y$ and $\sqrt{x} < \sqrt{y} \Rightarrow x < y$
   If $\sqrt{x}$ and $\sqrt{y}$ are irrational numbers, then, $x > \sqrt{y} \Rightarrow x^2 > y$ and $y > \sqrt{x} \Rightarrow y^2 > x$
2. If $a+b\sqrt{x} = c+d\sqrt{x} \Rightarrow a = c$ and $b = d$
3. The negative of an irrational number is always irrational.
4. The sum of a rational and an irrational number is always irrational.
5. The product of a non-zero rational number and an irrational number is always irrational.
6. The sum, difference, product and quotient of two irrational numbers may not be an irrational number.

Real Numbers

1. The set of the union of rational and irrational numbers forms a set of real numbers. Thus a real number can be associated with a point on the number line.
   $R = Q \cup \overline{Q}$



Surds

1. If x is a positive rational number and n is a positive integer such that $x^{\frac{1}{n}} = \sqrt[n]{x}$ is irrational, then $x^{\frac{1}{n}}$ is called a surd or a radical.
2. Every surd is an irrational number, but every irrational number is not a surd. π is an irrational number but it is not a surd.
3. When two irrational numbers are multiplied, their product is called a rational number. This process of removing the radical sign from the denominator is called rationalization and the two irrational numbers are called rationalizing factors of each other.
4. The irrational numbers $a + \sqrt{b}$ and $a - \sqrt{b}$ are called conjugates of each other.
5. A number of the form $\sqrt[n]{a}$ is called a surd of the order n where a is a positive rational number which is not equal to the n^{th} power of a rational number and n is a natural number greater than or equal to 2.