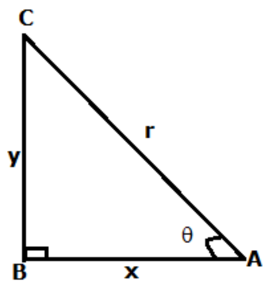
Trigonometrical Identities

Important Concepts

1. An angle which has a magnitude as well as the direction of rotation is known as directed angle.
2. An angle in the figure formed by two rays called initial ray and terminal ray, with a common initial point called vertex.
3. If the rotation from the initial ray to the terminal ray is clockwise it is to be taken as positive and if it is anti-clockwise, rotation is to be taken as negative.
4. Each trigonometrical ratio is a real number and has no units.

Important Formulae's

1. In a right angle triangle ABC, let $\angle ABC = \theta$



Let *AB=x*, *BC = y* and *AC = r.*

Then we define the trigonometric ratios as under

i. $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{y}{r}$

ii. $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{x}{r}$

iii. $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{y}{x}$

iv. $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{x}{y}$

v. $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{r}{x}$

vi. $\operatorname{cosec}\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{r}{y}$

2. $\sin\theta \times \operatorname{cosec}\theta = 1$
3. $\cos\theta \times \sec\theta = 1$
4. $\tan\theta \times \cot\theta = 1$
5. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
6. $\sin^2\theta + \cos^2\theta = 1$
7. $\operatorname{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow \cot^2\theta = \operatorname{cosec}^2\theta - 1$
8. $\sec^2\theta - \tan^2\theta = 1 \Rightarrow \tan^2\theta = \sec^2\theta - 1$

Relations between Trigonometric Ratios

1. Reciprocal relation:

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\sec A = \frac{1}{\cos A}$$

$$\tan A = \frac{1}{\cot A}$$

$$\cot A = \frac{1}{\tan A}$$

2. Quotient relation:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Fundamental Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow 1 - \sin^2\theta = \cos^2\theta \text{ and } 1 - \cos^2\theta = \sin^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\Rightarrow \sec^2\theta - \tan^2\theta = 1 \text{ and } \sec^2\theta - 1 = \tan^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1 \text{ and } \operatorname{cosec}^2\theta - 1 = \cot^2\theta$$

Trigonometrical Ratios of Complementary angles:

$$\sin\left(90^\circ - \theta\right) = \cos\theta$$

$$\cos\left(90^\circ - \theta\right) = \sin\theta$$

$$\tan\left(90^\circ - \theta\right) = \cot\theta$$

$$\cot\left(90^\circ - \theta\right) = \tan\theta$$

$$\sec\left(90^\circ - \theta\right) = \operatorname{cosec}\theta$$

$$\operatorname{cosec}\left(90^\circ - \theta\right) = \sec\theta$$

Using Trigonometric Tables:

To find the trigonometric ratios of acute angles other than $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$, we need to use the trigonometrical tables.

The trigonometric tables give the values of natural sines, cosines and tangents to four decimal places.
A trigonometrical table consists of three parts:

a. a column on the extreme left which contains degrees from 0° to 89°.
b. ten columns headed by $0', 6', 12', 18', 24', 30', 36', 42', 48'$ and $54'$ respectively.
c. five columns headed by $1', 2', 3', 4'$ and $5'$ respectively.

One degree (1°) is divided into sixty equal parts, each part is called one minute ($1'$).
That is, One degree = 60 minute.

Example:
Find $\sin 36^\circ 51'$

Solution:
Observe the table given for natural sines:

x°	$0'$	$6'\ 12'\ 18'$	$24'\ 30'\ 36'$	$42'\ 48'\ 54'$	$1'\ 2'\ 3'\ 4'\ 5'$ Difference to add
36	0.5878			5990	7

Since

$\sin 36^\circ 51' = \sin(36^\circ 48' + 3')$

From the table, we have

$\sin 36^\circ 48' = 0.5990$

diff for $3' = 0.0007$

Thus, $\sin 36^\circ 51' = 0.5997$